Algorithmic Game Theory

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Exercise Sheet 3

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Please hand in your solutions until Tuesday, November 12, 10:15h, in H9 or the letterbox between rooms 114 and 115, R.M.S. 11-15.

Exercise 3.1. (2+3 Points)

a) The bimatrix-game *battle of sexes* is defined by the following matrix.

	Zeil	Eintracht
Zeil	1 2	6 6
Eintracht	5 5	2 1

Write down an exact potential function for this game.

b) Construct a 2x2 bimatrix-game with a pure Nash equilibrium and without exact potential function. Prove that there is no exact potential function.

Exercise 3.2. (2+2+1 Points)

a) For which α does the function $d_r : \{1, \ldots, n\} \to \mathbb{N}$ with $d_r(x) = x^2$ fulfill the α-bounded jump property?

b) Let $d : \{1, \ldots, n\} \to \mathbb{N}$ be a positive, monotonically increasing function with *bounded slope*, i.e.

$$|d(x_1) - d(x_2)| \leq K \cdot |x_1 - x_2|$$

for some constant K. Does it fulfill the $(K+1)$-bounded jump property?

c) Let $d : \{1, \ldots, n\} \to \mathbb{N}$ be a positive, monotonically increasing function with $(K+1)$-bounded jump property. Is it true that

$$|d(x_1) - d(x_2)| \leq K \cdot |x_1 - x_2|$$

for all $x_1, x_2 \in \{1, \ldots, n\}$?

Exercise 3.3. (4 Points)

A weighted congestion game is called a singleton game if $|S_i| = 1$ for all $i \in N$ and $S_i \in \Sigma_i$. Prove that singleton weighted congestion games with strictly increasing delay functions d_r are ordinal potential games.

Hint: Lexicographical decrease.

Exercise 3.4. (4 Points)

Finish the proof from the lecture and show that weighted congestion games with affine delay functions have a pure Nash equilibrium.

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