Algorithmic Game Theory

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Exercise Sheet 4

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Please hand in your solutions until Tuesday, November 19, 10:15h, in H9 or the letterbox between rooms 114 and 115, R.M.S. 11-15.

Exercise 4.1. (3 + 2 + 2 Points)

For this exercise, we consider the special case of a matching game with the same number of men and women, i.e., $|\mathcal{X}| = |\mathcal{Y}|$. Note that there are matching games in which there exist multiple stable matchings. We define the term *feasible partner* as follows.

Definition 1. *Let $x \in \mathcal{X}$ be a man. A woman $y \in \mathcal{Y}$ is called a* feasible partner *of x if there is a stable matching in which x and y are matched together.*

The definition for women is analogously.

a) Show that the Deferred Acceptance Algorithm with man proposal matches every man to his most preferred feasible partner.

b) Show that the Deferred Acceptance Algorithm with man proposal matches every woman to her least preferred feasible partner.

c) Design an algorithm that decides in polynomial time if the stable matching is unique. What is the asymptotic running time of your algorithm?

Exercise 4.2. (2 + 1 + 1 + 3 Points)

Consider the following 2-player bimatrix game.

	E	F	G
A	10 10	10 100	100 1
B	100 10	10 10	1 10
C	1 1	10 100	10 10

The game is played repeatedly. Suppose the players choose the following sequence of strategies:

$$\binom{A}{E}, \binom{B}{F}, \binom{C}{G}, \binom{A}{E}, \binom{B}{F}, \binom{C}{G} \cdots$$

a) Show that this sequence fulfills the no-regret property for both players.

b) Let $\mathcal{V}$ be a probability distribution over the states defined as follows:

$$Pr(s) = \begin{cases} \frac{1}{3} & \text{if } s = \binom{A}{E} \\ \frac{1}{3} & \text{if } s = \binom{B}{F} \\ \frac{1}{3} & \text{if } s = \binom{C}{G} \\ 0 & \text{else.} \end{cases}$$

Is $\mathcal{V}$ a coarse correlated equilibirum? Why or why not?

c) Is the average strategy a mixed Nash equilibrium? Prove your statement.

d) Modify the game such that the given sequence fulfills property a), does not fulfill property c), and at least one of the strategies that is played is strictly dominated. Justify your solution.

Exercise 4.3. (2 + 2 + 2 Points)

For the analysis of the randomized weighted majority algorithm, we considered an adversary that generated the cost of the experts $\ell^1, \ell^2, \dots, \ell^T$. This shows that the analyzed algorithm meets the no-regret property, even if the costs are generated in a different (non adversarial) way. Even for the adversarial model there are different versions regarding the knowledge and the power of the adversary. Consider the following three cases.

Oblivious Adversary: All cost vectors of the experts, $\ell^1, \ell^2, \dots, \ell^T$, are generated and fixed before round 1 and the first decision of the algorithm. Vector ℓ^t is only presented to the algorithm in round t.

Adaptive Online Adversary: In every round t, the adversary knows the probability distribution of the algorithm for choosing an expert. The choice of ℓ^t is based on this knowledge.

Adaptive Offline Adversary: In every round t, the adversary knows the expert that is chosen (after a random draw according to the probability distribution) by the algorithm. Based on that, the adversary chooses a cost vector ℓ^t in round t.

Argue for all three models, whether or not there exists an algorithm with no-regret guarantee for the Expert Problem.

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