Algorithmic Game Theory

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Prof. Dr. Martin Hoefer, Dr. Daniel Schmand

JOHANN WOLFGANG GOETHE UNIVERSITÄT FRANKFURT AM MAIN

Algorithmen und Komplexität
Institut für Informatik

Exercise Sheet 5

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Solutions Due: Nov 26, 2019

Please hand in your solutions until Tuesday, November 26, 10:15h, in H9 or the letterbox between rooms 114 and 115, R.M.S. 11-15.

This exercise sheet contains a programming exercise. Please hand in the specified output of your program by Tuesday 10:15. Additionally, present and explain the working algorithm to Daniel latest until Thursday, November 28, 18:00. You do not need to hand in your code.

Exercise 5.1. (4 or 0 Points)

Implement the randomized weighted majority algorithm in your favorite programming language. Input to your algorithm is a parameter T specifying the number of rounds and a normalized 2×2 matrix that specifies the utility of player 1 in a two-player zero-sum game. Given this game, both players start in round 1 with strategies $(0.5, 0.5)$ and both perform the randomized weighted majority algorithm to determine the strategies for the next $T-1$ rounds. Use $\eta = \sqrt{\frac{ln2}{T}}$ for both players. Note that the values l_i^t for some player are always specified by the randomized strategy of the other player in the last round. For simplicity, we always work with probabilities, i.e. we immediately rescale the weights w_1^t and w_2^t such that $w_1^t + w_2^t = 1$. Your software should be able to print the weights for both players after each round.

Please hand in the output of your algorithm for the following test cases by Tuesday, 10:15.

a) T=3, A=$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (Print the probabilities after rounds 1,2,3.)

b) T=5, A=$\begin{bmatrix} 0.7 & 0.3 \\ 0 & 1 \end{bmatrix}$ (Print the probabilities after rounds 1,2,3,4,5.)

c) T=300, A=$\begin{bmatrix} 1 & 0.5 \\ 0.2 & 0 \end{bmatrix}$ (Print the probabilities after rounds 100,200,300.)

Additionally, present and explain your code to Daniel by Thursday, November 28, 18:00.

Exercise 5.2. (3 Points)

Show that in a finite game Γ every mixed Nash equilibrium is a coarse-correlated equilibrium in the following sense: Given a mixed Nash equilibrium $(x_{ij})_{i \in N, j \in S_i}$, we define the distribution over states $\mathcal{V}$ by

$$p(s) = \prod_{i \in N} x_{i,s_i} \ .$$

Prove that $\mathcal{V}$ is a coarse-correlated equilibrium.

Exercise 5.3. (4 Points)

In the olive oil game, there are n companies that can all produce olive oil in the same quality. Each company i has an upper bound of b_i liters on the production capacity. This upper bound is naturally given by the size and the amount of available olives and oil presses to company i. The exact production $s_i \in [0, b_i]$ (specified in liters) is a strategic decision of company i. The production cost of company i for s_i liters of olive oil is given by $s_i \cdot c_i$ for some constant $c_i > 0$. The price per liter olive oil on the market is the same for all companies and given by the affine-linear function $d - a \cdot \sum_{i=1}^{n} s_i$, thus it is linear decreasing in the total amount of available olive oil. The profit of company i in state s is given by

$$u_i(s) = s_i \left(d - a \sum_{i=1}^{n} s_i \right) - s_i c_i \ .$$

Show that the olive oil game is a socially concave game.

Email: `mhoefer@cs.uni-frankfurt.de, schmand@em.uni-frankfurt.de`