Algorithmic Game Theory

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Exercise Sheet 7

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Please hand in your solutions until Tuesday, December 10, 10:15h, in H9 or in the letterbox between rooms 114 and 115, R.M.S. 11-15.

Exercise 7.1. (3 Points)

Give an example for an equal-sharing game for which the best Nash equilibrium does not minimize the potential function. Argue why your example has the desired properties.

Exercise 7.2. (3 + 4 Points)

a) Construct a congestion game with linear delay functions and price of stability of at least 1.5. Argue why your example has the desired properties.

b) Prove that the price of stability in congestion games with linear delay functions is at most 2.

Exercise 7.3. (3 + 3 (+3) Points)

a) Design an ordinal potential function for weighted temporal congestion games with the shortest-first-policy on parallel links. I.e., you can assume that the network only consists of 2 vertices s and t and parallel edges (s, t) with possibly different speeds. Argue that your function is an ordinal potential function.

b) We generalize the game in the following sense. Every player i has a release date r_i and she cannot be scheduled on any edge before r_i. We still use the shortest-first policy without preemption, i.e. if at some time τ an edge e is available, it non-preemptively processes the highest ranked player among those that have not been processed by e, have e in their strategy, and $r_i \leq \tau$. Is your function from a) still an ordinal potential function?

c) Bonus: Do games as defined in b) always have a pure Nash equilibrium?

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