Algorithmic Game Theory

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Exercise Sheet 10

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Please hand in your solutions until Tuesday, January 28, 10:15h, in H9 or in the letterbox between rooms 114 and 115, R.M.S. 11-15.

Exercise 10.1. (2 Points)

Prove that the plurality social choice function does not always fulfill the condorcet-winner condition (CWC).

Exercise 10.2. (2 Points)

Show that a social choice function is incentive compatible (IC) if and only if it is monotone.

Exercise 10.3. (2 Points)

Show that there is no social choice function for three or more alternatives that satisfies the always-a-winner (AAW), independence of irrelevant alternatives (IIA) and the condorcet-winner condition (CWC).

Exercise 10.4. (2 Points)

Let n be odd. Show that the Median-rule for single-peaked preferences fulfills the condorcet-winner condition (CWC).

Exercise 10.5. (2+2+2+2+2 Points)

Consider a beach that can be represented by the interval $[0, 1]$. There are n people visiting the beach and visitor i has a most favorite spot $s_i \in [0, 1]$. We would like to place ice-cream sellers at the beach. We ask every visitor for the most preferred position $b_i \in [0, 1]$ and each visitor i has an incentive that an ice-cream seller gets placed as close as possible to s_i. Let $b = (b_1, \dots, b_n)$. First, assume that we only place a single ice-cream seller at position $p_1 \in [0, 1]$.

a) Let $d^{\Sigma}(p_1, b)$ be the total distance of all reported positions to the ice-cream seller at p_1, i.e.,

$$d^{\Sigma}(p_1, b) = \sum_{i=1}^{n} |b_i - p_1| \ .$$

Is there an incentive-compatible mechanism without money such that $d^{\Sigma}(p_1, b)$ is minimized? Prove your answer.

b) Consider the maximum distance of any visitor to p_1, i.e.,

$$d^{\max}(p_1, b) = \max_{i \in \mathcal{N}} |b_i - p_1| \ .$$

Is there an incentive-compatible mechanism without money such that $d^{\max}(p_1, b)$ is minimized? Prove your answer.

For the following tasks, assume we place two ice-cream sellers at $p_1, p_2 \in [0,1]$.

c) Consider again the maximum distance of any visitor to the next ice-cream seller. Let

$$d^{\max}(p_1, p_2, b) = \max_{i \in \mathcal{N}} \{\min(|b_i - p_1|, |b_i - p_2|)\} .$$

Is there an incentive-compatible mechanism without money such that $d^{\max}(p_1, p_2, b)$ is minimized? Prove your answer.

d) Consider the following max-min-mechanism: Choose $p_1 = \max_i b_i$ and $p_2 = \min_i b_i$. Is this mechanism incentive compatible?

e) Prove that the max-min-mechanism is a 2-approximation for the maximum distance, i.e.,

$$d^{\max}(p_1, p_2, b) \quad \leq \quad 2 \cdot \min_{q_1, q_2 \in [0,1]} d^{\max}(q_1, q_2, b) .$$