Algorithmic Game Theory

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Assignment 5

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Due: Nov 29, 2022, **10:00h**

Exercise 5.1. (2 + 1 + 1 + 3 Points)

Consider the following 2-player bimatrix game.

	E	F	G
A	10, 10	100, 10	1, 100
B	10, 100	10, 10	10, 1
C	1, 1	100, 10	10, 10

The game is played repeatedly. Suppose the players choose the following sequence of strategies:

$$\binom{A}{E}, \binom{B}{F}, \binom{C}{G}, \binom{A}{E}, \binom{B}{F}, \binom{C}{G}, \ldots$$

a) Show that this sequence fulfills the no-regret property for both players.

b) Prove or disprove: The average strategy of the given sequence converges to a mixed Nash equilibrium.

c) Let $\mathcal{V}$ be a probability distribution over states, such that for $s \sim \mathcal{V}$ and every state $\tilde{s}$, it holds:

$$\mathbb{P}(s = \tilde{s}) = \begin{cases} \frac{1}{3} & \text{if } \tilde{s} \in \left\{ \binom{A}{E}, \binom{B}{F}, \binom{C}{G} \right\} \\ 0 & \text{else.} \end{cases}$$

Prove or disprove: $\mathcal{V}$ is a coarse-correlated equilibrium.

d) Modify the game such that all of the following requirements are met:

- The given sequence fulfills the no-regret property for both players.
- The average strategy of the given sequence doesn't converge to a mixed Nash equilibrium.
- At least one of the strategies is strictly dominated.

Justify your solution.

Exercise 5.2. (4 Points)

The Expert problem from the lecture assumes that the total number of time steps T is known. The RWM algorithm exploits this by choosing the parameter η depending on T to achieve the no-regret property. However, every no-regret algorithm A also works in environments with unknown T with a slight modification:

We split the process into an unknown number of phases, starting with phase 0. For every $k \geq 0$, phase k consists of steps $2^k, \dots, 2^{k+1} - 1$ and therefore 2^k steps in total. At the beginning of each phase, (re)start A for $T' = 2^k$ time steps. If the last time step was reached and no new round occurs, stop A. All remaining steps of that last phase are assumed to yield no costs.

Assume that A has a regret of at most $\alpha\sqrt{T}$ when T is known, on any sequence of length T. Prove that the modified algorithm described above has a regret of at most $\frac{\sqrt{2}}{\sqrt{2}-1} \cdot \alpha\sqrt{T}$.

Exercise 5.3. (3 + 2 + 2 Points)

Consider a matching game with the same number of men and women, i.e., $|\mathcal{X}| = |\mathcal{Y}|$. Note that there are matching games in which multiple stable matchings exist.

Let $x \in \mathcal{X}$ be a man in a matching game. A woman $y \in \mathcal{Y}$ is a *feasible partner* for x if there is a stable matching in which x and y are matched. Feasible partners for women are defined analogously.

a) Show that the Deferred Acceptance Algorithm with man proposal matches every man to his most preferred feasible partner.

b) Show that the Deferred Acceptance Algorithm with man proposal matches every woman to her least preferred feasible partner.

c) Design an algorithm that decides in polynomial time if there is a unique stable matching. Explain the exact asymptotical running time and argue why your algorithm is correct.