Algorithmic Game Theory

Winter Term 2022/2023

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Assignment 8

Issued: Dec 13, 2022
Due: Dec 20, 2022, **10:00h**

This and all upcoming assignments are only relevant for the second part of the course.

Exercise 8.1. (2 + 2 Points)

There is an auction with one good and $n > 2$ bidders with private valuations for the good. We assume that their bids are pairwise distinct. The good is assigned to the bidder with the highest bid, and the price is p. The other bidders neither receive nor pay anything.

State all values of n for which the described mechanism is incentive compatible, when p is

a) the arithmetic mean of all bids,

b) the (lower) median bid (always take the $\lfloor \frac{n+1}{2} \rfloor$-th smallest bid).

Prove your answers.

Exercise 8.2. (4 Points)

We consider a combinatorial auction: There is a set $\mathcal{G}$ of goods that are indivisible. Bidders have a private valuation for each subset of $\mathcal{G}$. Every outcome allocates exactly one subset of $\mathcal{G}$ to every bidder such that the intersection of each pair of allocated subsets is empty.

Let $\mathcal{G} = \{A, B, C\}$ and $\mathcal{N} = \{1, 2, 3, 4\}$. The private valuations $v_i(G)$ for all bidders $i \in \mathcal{N}$ and subsets $G \subseteq \mathcal{G}$ are given by the following table:

	$\emptyset$	$\{A\}$	$\{B\}$	$\{C\}$	$\{A,B\}$	$\{A,C\}$	$\{B,C\}$	$\{A,B,C\}$
$i=1$	0	2	3	2	2	4	6	6
$i=2$	0	0	0	3	0	6	5	9
$i=3$	0	1	5	1	4	1	1	2
$i=4$	0	1	2	3	3	5	3	5

Apply the VCG mechanism on this instance: Explain which allocation is chosen by VCG, and why. Then calculate the payments for all bidders (with explanation). You may assume true bidding.

Exercise 8.3. (3 + 3 + 2 Points)

For a given undirected graph $G = (V, E)$ with edge capacities $c_e \in \mathbb{N}$ for all $e \in E$, we want to auction matchings. Each bidder $i \in \mathcal{N}$ has a desired matching $M_i \subseteq E$, a demand $d_i \in \mathbb{N}$ and a private valuation $v_i \in \mathbb{R}_+$ when they are selected. Otherwise, the valuation is 0.

After seeing all bids, the auctioneer selects a subset $\mathcal{I} \subseteq \mathcal{N}$ of bidders and decides on the prices for all bidders. Let M be the union of matchings from selected bidders, i.e. $M = \bigcup_{i \in \mathcal{I}} M_i$. The selection of bidders $\mathcal{I}$ needs to be feasible in the following sense:

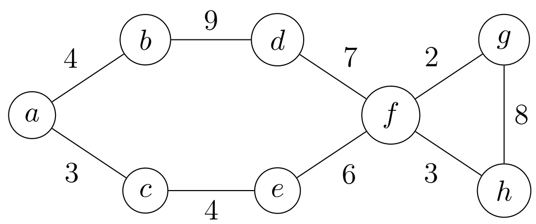
- M has to be a matching itself.
- For each $e \in M$, the sum of demands of all $i \in \mathcal{I}$ with $e \in M_i$ does not exceed c_e.

A selection rule for this auction when receiving bids $b_1, \ldots, b_n$ is given as follows: We start with $\mathcal{I} = \emptyset$ and remove bidders with $M_i = \emptyset$ or $b_i = 0$ beforehand. Then:

- Sort bidders by their reported value per demand per edge, i.e. by $\frac{b_i}{d_i \cdot |M_i|}$. Break ties arbitrarily.
- Iterate over the ordered bidders and greedily add bidder i into $\mathcal{I}$, unless the selection becomes infeasible due to i.

a) Prove that the given selection rule can be used by an incentive compatible mechanism. Construct such mechanism, i.e. state a suitable payment formula.

b) Consider the following instance with 4 bidders:



i	M_i	d_i	v_i
1	$\{\{a,b\},\{c,e\},\{g,h\}\}$	3	27
2	$\{\{d,f\},\{g,h\}\}$	5	30
3	$\{\{a,c\},\{b,d\},\{e,f\},\{g,h\}\}$	3	24
4	$\{\{c,e\},\{d,f\}\}$	2	10

Calculate the selection and the prices for all bidders when the mechanism from a) is applied.

Hint: Pictures might be helpful when explaining your calculation, which is required!

c) Add new bidders to the instance from b) such that the following requirements are met:

- There are at most two additional bidders.
- The selection retrieved by the mechanism from a) is not socially optimal.

Prove the correctness of your modification.

Assignments and further information concerning the course can be found at
`http://algo.cs.uni-frankfurt.de/lehre/agt/winter2223/agt2223.shtml`

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