Algorithmic Game Theory

Winter Term 2022/2023

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Assignment 9

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Due: Jan 17, 2023, **10:00h**

We wish you happy holidays and a great start into 2023!

Exercise 9.1. (2 + (1 + 1) Points)

The city council suffered a lot of criticism in previous years for their choice of the Christmas tree in front of the city hall. This year the residents of the city are involved in the decision process, but they also have to contribute directly to financing the Christmas tree. The choice is between the usual standard tree S, a very fancy but more expensive tree F, or no tree at all ($\emptyset$), without cost.

Each of the n residents of the city has a private valuation $v_i(A) \geq 0$ for $A \in \{\emptyset, S, F\}$. The final choice should maximize social welfare while taking the cost into account. Formally, the city council acts as an additional player with a public valuation of $-C_S$ for the standard tree, and a public valuation of $-C_F$ for the fancy tree. As the city council promised to subsidize the choice, the payment for this additional player does not have to be considered. If there is not a Christmas tree at all, the valuation of everyone is 0. It holds $C_F > C_S > 0$.

a) Design an incentive compatible mechanism $(f, p_1, \dots, p_n)$ with no positive transfers to residents for the described situation. Provide f and $p_1, \dots, p_n$ formally.

b) Prove or disprove: For any n, C_S, C_F, there is always an instance such that the mechanism from a) chooses an outcome with the following properties:

- The city decides to have a Christmas tree this year, i.e. the outcome is either S or F.
- The total payments of all residents either
  i) exactly cover the cost of the chosen Christmas tree, or
  ii) are 0, i.e. the Christmas tree is entirely payed for by the city council.

Exercise 9.2. (3 Points)

For Myerson's Lemma, we showed in a *proof by picture* that a monotone, piecewise constant allocation rule x with the payment formula

$$p_i(v_i, v_{-i}) = v_i \cdot x_i(f(v)) - \int_{t_i^0}^{v_i} x_i(f(t, v_{-i}))dt$$

yields an incentive compatible mechanism.

Give an example for a non-monotone allocation function such that truthful bidding is not a dominant strategy for at least one player when the payment formula above is used. Explain where the proof by picture breaks down.

Exercise 9.3. (3 + 3 Points)

We stated that the standard FPTAS for knapsack with granularity parameter $s = \varepsilon \cdot v_{\max}/n$ is not necessarily monotone.

a) Prove this statement by giving an example where the output S of that FPTAS is not monotone for at least one bidder.

b) Prove that this scheme is monotone for all bidders if $s = \delta > 0$ independent of $v_1, \ldots, v_n$ is chosen as granularity parameter.

Exercise 9.4. (3 + 3 Points)

Consider an auction with k identical items. Each bidder is only interested in getting one of the items. The seller wants to get at least a total revenue of $R \geq 0$. The mechanism is defined as follows:

Collect the bids $b_1, \ldots, b_n$.
Initialize the set S with the k highest bidders, set payments $p_i = 0$ for all $i \in [n]$.
while *there is a bidder* $i \in S$ *with* $b_i < R/|S|$ **do**
    Delete such a bidder from S.
if $S \neq \emptyset$ **then**
    for *all bidders* $i \in S$ **do**
        allocate one item to player i and set $p_i = R/|S|$.

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a) Show the following: Let M be any normalized, and incentive compatible mechanism with the property that all players that get an item have to pay the same. If M generates a revenue of at least R, than the mechanism above also generates a revenue of at least R.

b) Show the following: There is an example for an incentive compatible mechanism M that guarantees a revenue of at least R, where the mechanism above does not generate a revenue of R. (In this case, M does not have the property that all players that get an item have to pay the same.)

Exercise 9.5. (4 Points)

Consider the VCG auction with k identical items, for the case that every bidder only wants to get one item. (Allocate k items to the k highest bidders, everyone of these pays the $(k + 1)$-highest bid). Can you extend the theorem of Bulow-Klemperer to this case? If it is possible, which is the minimum number x of additional bidders such that the VCG auction with $n+x$ bidders guarantees at least the same expected revenue as the one with the optimal mechanism with n bidders? Prove your answers.

Assignments and further information concerning the course can be found at
`http://algo.cs.uni-frankfurt.de/lehre/agt/winter2223/agt2223.shtml`

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