Algorithmic Game Theory

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Assignment 8

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Due: Jan 15, 2024, **23:55h**

Exercise 8.1. (2 + 3 + 3 Points)

a) Consider the Greedy algorithm for knapsack auctions described in the lecture. Show that Greedy is monotone for every bidder.

b) We stated in the lecture that the FPTAS with granularity parameter $s = \varepsilon \cdot v_{\max}/n$ is not necessarily monotone. Prove this statement, i.e., give an example, where the allocation defined by the FPTAS is not monotone for at least one bidder.

c) Prove that the same scheme with $s = \delta > 0$ independent of $v_1, \dots, v_n$ is monotone for all bidders.

Exercise 8.2. (2 + 3 Points)

Consider an auction for n bidders and a set M of different goods. Each bidder is only interested in a certain subset $T_i \subseteq M$ of the goods. The subsets T_i are publicly known, but the corresponding value v_i of the subset is private.

The mechanism queries all private values. It determines a result, i.e., an overlap-free allocation of goods to bidders (with $S_i \cap S_j = \emptyset$ for $i \neq j$ and $\bigcup_i S_i \subseteq M$) and payments p_i. The utility of bidder i is given by $v_i \cdot x_i - p_i$ where $x_i = 1$ if $S_i \supseteq T_i$ and 0 otherwise.

Consider the following Greedy algorithm:

Collect the bids $b_1, \dots, b_n$.
Initialize the set W as empty set, set $X = M$.
Sort bidders: $b_1 \geq b_2 \geq \cdots \geq b_n$.
for $i = 1, 2, 3, \dots, n$ **do**
  if $T_i \subseteq X$ **then**
    Remove T_i from X and add i to W.
return $S_i = T_i$ for $i \in W$, $S_i = \emptyset$ otherwise.

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a) Prove or disprove: The social choice function is monotone.

b) Let $d = \max_i |T_i|$. We denote the optimal allocation by S^*. Further, W^* contains all bidders who receive their subset of goods with respect to S^*, i.e., $S_i^* = T_i \quad \forall i \in W^*$.

   Show that Greedy is a d-approximation algorithm, i.e.,

$$\sum_{i \in W} v_i \geq \frac{1}{d} \cdot \sum_{i \in W^*} v_i \ .$$

   Hint: If Greedy selects a "suboptimal" bidder i, how many "optimal" bidders can i block?

Exercise 8.3. (2 + 2 + 2 Points)

Consider a single-item auction with two bidders.

a) Assume $\mathcal{V}_1$ is uniform over $[0,1]$ and $\mathcal{V}_2$ is uniform over $[0,2]$. Derive the virtual value functions for both bidders and calculate the winner of an optimal auction and corresponding payments, for $v_1 = 1$ and $v_2 = 5/3$.

b) Show that in an optimal auction, the highest bidder may not win the item (even though the highest bidder has a positive virtual value).

c) Give an intuitive explanation why the property shown in b) is beneficial in terms of revenue.

Exercise 8.4. (3 + 3 Points)

Consider an auction with k identical items. Each bidder is only interested in getting one of the items. The seller wants to get at least a total revenue of $R \geq 0$. The mechanism is defined as follows:

Collect the bids $b_1, \ldots, b_n$.
Initialize the set S with the k highest bidders, set payments $p_i = 0$ for all $i \in [n]$.
while *there is a bidder* $i \in S$ *with* $b_i < R/|S|$ **do**
    Delete such a bidder from S.
if $S \neq \emptyset$ **then**
    for *all bidders* $i \in S$ **do**
        Allocate one item to player i and set $p_i = R/|S|$.

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a) Show the following: Let M be any normalized, and incentive compatible mechanism with the property that all players that get an item have to pay the same. If M generates a revenue of at least R, than the mechanism above also generates a revenue of at least R.

b) Show the following: There is an example for an incentive compatible mechanism M that guarantees a revenue of at least R, where the mechanism above does not generate a revenue of R. (In this case, M does not have the property that all players that get an item have to pay the same.)

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