Approximation Algorithms

Winter term 2021/22

Prof. Dr. Martin Hoefer
Tim Koglin, Lisa Wilhelmi

GOETHE UNIVERSITÄT FRANKFURT AM MAIN
Institute of Computer Science
Algorithms and Complexity

Assignment 1

Issued: 26.10.2021
Due: 02.11.2021, **10:15h**

- To participate in the exercises join our course on moodle. The link is also provided on our website (see footer).
- Solutions must be submitted as **one PDF-file via moodle**.
- Make sure that your submission reaches us **before 10:15h**. Please take your upload time into account which depends on your internet connection and load on moodle.
- This assignment will be discussed in the first exercise session on **November 5th** (online, via Zoom). Your submission will be reviewed and returned to you via moodle beforehand.
- Submissions can be composed in English or German.
- Your submission must include your **name** and **student ID number**.
- As a general rule, you need to **explain** your answers to get full points if not stated otherwise.

We wish you a good semester!

Exercise 1.1 NP-hardness of Makespan Scheduling (5 points)

In the NP-hard Partition problem, a finite, non-empty multiset A of natural numbers is given. The problem is to decide whether or not A can be subdivided into two subsets A_1, A_2, where $A_1 \cup A_2 = A$ and $A_1 \cap A_2 = \emptyset$, such that $\sum_{x \in A_1} x = \sum_{y \in A_2} y$.

Prove that the decision version of the Makespan Scheduling problem is NP-hard by showing Partition $\leq_p$ Makespan Scheduling.

Exercise 1.2 *Coloring Edges* (5 points)

The min-Edge-Coloring problem is defined as follows: An undirected graph $G = (V, E)$ with node set V and edge set E is given. The goal is to color the edges of G using as few colors as possible such that no two edges of the same color are incident to a common node. Let $OPT(G)$ denote the minimum number of different colors needed for coloring the edges of G.

Show that there exists a Greedy algorithm that needs at most $2 \cdot OPT(G) - 1$ different colors for any graph G. Prove that your algorithm always obtains a valid solution, i.e., no two edges of the same color are incident to a common node.

Exercise 1.3 *Algorithms for* MAKESPAN SCHEDULING (1 + 2 + 2 points)

For the MAKESPAN SCHEDULING problem, the *MinSort* algorithm operates as follows: First, the tasks $1, ..., n$ are sorted in non-decreasing order according to their processing times $p_i > 0$. After that, the *ListScheduling* algorithm from the lecture is performed on this generated permutation.

a) Determine the approximation ratio of *MinSort* for the following instance: Let the number of machines be $m = 35$ and the number of jobs be $n = 103$. Assume that the processing time of 99 of these jobs is $p_i = 2$, and $p_j = 3$ is the processing time of the 4 remaining jobs.

b) Construct a worst-case instance for *MinSort* for $m = 2$ machines and state the resulting approximation ratio. It is not necessary to prove that there is no worse instance.

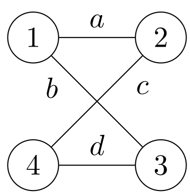
c) Consider the algorithm *MaxSort* which operates in the same manner as *MinSort* but sorts the tasks in non-increasing order.

   Construct a worst-case instance for *MaxSort* for $m = 2$ machines and state the resulting approximation ratio. It is not necessary to prove that there is no worse instance.

Exercise 1.4 *Matroids* (1 + 2 + 2 points)

a) Consider the undirected graph $G = (V, E)$ depicted below, where $V = \{1, 2, 3, 4\}$ and $E = \{a, b, c, d\}$. In the graphic matroid $(\mathcal{T}, E)$, the set $\mathcal{T}$ includes all subsets of edges that form a forest in G.

   State the set of independent sets $\mathcal{T}$ of the matroid and its rank. No explanation is required.



b) Consider a *directed* graph $G = (V, E)$ where a number $k_v \in \mathbb{Z}_{>0}$ is assigned to any node $v \in V$. A set system $(\mathcal{D}, X)$ is defined by $X := E$ and $\mathcal{D} := \{I \subseteq E \mid \text{indeg}_I(v) \leq k_v \text{ for all } v \in V\}$. Here, $\text{indeg}_I : V \to \mathbb{N}_0$ is the in-degree of a node with respect to the set of edges I, i.e., $\text{indeg}_I(v) := |\{u \in V \mid (u, v) \in I\}|$.

   Show that $(\mathcal{D}, X)$ is a matroid.

c) Now let $G = (V, E)$ be an *undirected* graph. Analogous to b), let the set system $(\mathcal{U}, X)$ be defined by $X := E$ and $\mathcal{U} := \{I \subseteq E \mid \deg_I(v) \leq k_v \text{ for all } v \in V\}$, where $\deg_I : V \to \mathbb{N}_0$ is the degree of a node with respect to the set of edges I, i.e., $\deg_I(v)_= |\{u \in V \mid \{u, v\} \in I\}|$.

   Show that the pair $(\mathcal{U}, X)$ is not a matroid in general by giving a counter-example.

The assignments and further information on the course are provided on our website:
https://algo.cs.uni-frankfurt.de/lehre/apx/winter2122/apx2122.shtml

Contacts of tutors:: {koglin,wilhelmi}@em.uni-frankfurt.de.