Approximation Algorithms

Winter term 2021/22

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Assignment 12

Issued: 08.02.2022
Due: 15.02.2022, **10:15h**

- This is the last assignment of the course. Its scope is halved.
- Exercises marked with * are bonus - they count for your score but not for the sum of points.

Exercise 12.1 *Submodular Functions* (4 + 2* points)

a) For each of the following problems on an undirected graph $G = (V, E)$, decide whether the set function $f(S)$, where $S \subseteq V$, is submodular or not.

(i) For every set S, the function f returns the cardinality of a maximum independent set I such that $I \subseteq S$.

(ii) Consider weights $w_e > 0$, for all edges $e \in E$, and two vertices $s, t \in V$ as given. For every s-t-cut S, let the function f be defined as the overall cost of the cut, i.e., $f(S) = \sum_{e\in\delta(S)} w_e$.

b)* For the functions defined in (i) and (ii), decide whether they are monotone or not.

Exercise 12.2 *Cycles and Superstrings* (3 + 3 points)

a) Consider the *greedy superstring* algorithm for Shortest Common Superstring discussed in the lecture. Given $\varepsilon > 0$, construct an instance U consisting of three strings over a finite alphabet Σ such that *greedy superstring* computes a $(3/2 - \varepsilon)$-approximate solution, where for each pair of strings $u, v \in U$, u must not be a substring of v.

b) Assume four strings u, v, u^*, v^* satisfying

$$\text{overlap}(u, v) \geqslant \text{overlap}(u, v^*) \quad \text{and} \quad \text{overlap}(u, v) \geqslant \text{overlap}(u^*, v) \ .$$

Show that

$$\text{overlap}(u, v) + \text{overlap}(u^*, v^*) \geqslant \text{overlap}(u, v^*) + \text{overlap}(u^*, v) \ .$$