Optimization and Uncertainty

Summer term 2021

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Assignment 1

Issued: 20.04.2021
Due: 27.04.2021, **10:00h**

- Solutions have to be submitted as **one PDF-file via e-mail** to Lisa Wilhelmi.
- Make sure that your submission reaches us **before 10:00h**.
- Submissions can be composed in English or German.
- Your submission must include your **name** and **matriculation number**.
- In general, you need to **explain** your answers if not stated otherwise.
- This assignment will be discussed in the exercise session on April 30th.

We wish you a good semester!

Exercise 1.1 *Lower bounds for* OnlineMax (3 points)

Consider the OnlineMax problem discussed in the lecture. Show that no deterministic algorithm is α-competitive, for any finite α.

Exercise 1.2 *Online*-Secretary $(1+1+1+1=4$ points)

Consider the optimal algorithm for the Secretary problem from the lecture. Suppose that for some job there are 150 candidates in total. After interviewing the first 40 candidates, the first candidate is accepted whose value exceeds the highest value among the first 40 candidates. If there is no such candidate, the last candidate is accepted.

a) Determine the probability that the best candidate shows up last.

b) Determine the probability that the best candidate shows up last **and** that this candidate is accepted.

c) Determine the probability that the last candidate is accepted.

d) Determine the **conditional probability** that the last candidate is accepted given that this is the best candidate.

Exercise 1.3 *Strategies for the* Secretary *problem* $(3+3=6$ points)

Compute and compare the probability of accepting the best candidate of the two following strategies for the Secretary problem with n candidates.

a) Accept candidate at position $n-1$ if this candidate has a higher value than all previous ones. Otherwise accept the last candidate.

b) Accept candidate at position $n-2$ if this candidate has a higher value than all previous ones. Otherwise accept the last candidate.

Exercise 1.4 *Chernoff Inequality* (2 + 3 = 5 points)

a) An unfair coin turns up heads with probability $\frac{4}{5}$ and tails with probability $\frac{1}{5}$. Using the Chernoff inequality, estimate the upward probability that tails turns up at least five times in a sequence of ten coin flips.

Hint: Use the definition of the inequality in the appendix of the lecture notes:

$$\Pr[X \geq (1+\delta) \cdot \mathbb{E}[X]] \leq e^{-\mathbb{E}[X] \cdot \delta^2/3} .$$

b) Consider an unfair coin with probabilities $\frac{4}{5}$ and $\frac{1}{5}$. Unfortunately, you forgot whether heads or tails is more likely. Depict a method that assings the probabilities to the events with n coin tosses and an error probability of at most $e^{-\mathcal{O}(n)}$.