Optimization and Uncertainty

Summer term 2021

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Assignment 2

Issued: 27.04.2021
Due: 04.05.2021, **10:00h**

Exercise 2.1 *Independent probabilities* (4 points)

A (not necessarily fair) coin shows heads with probability p and tails with probability $1-p$ at each toss. Assume the coin is tossed two times in a row. Determine all values $p \in [0, 1]$ for which the two following events are independent.

A: Both tosses yield the same outcome.

B: The second toss yields heads.

Exercise 2.2 *Greedy Matching* (6 points)

Consider a graph $G = (V, E)$. Every edge $e \in E$ has a value $v_e \geq 0$. A *matching* is a set of edges $M \subseteq E$ where any node $u \in V$ is incident to not more than one edge in M, i.e., $|\{e \in M, u \in e\}| \leq 1$. The value of M is defined by $v(M) = \sum_{e \in M} v_e$.
Let M^* be a matching with maximum value. The *GreedyAlgo* starts with $M_g = \emptyset$ and iterates over all edges consecutively in non-ascending order of their values. An edge is added to M_g if M_g is still a matching thereafter. Show that the resulting greedy matching M_g satisfies

$$v(M_g) \geq \frac{1}{2} \cdot v(M^*) \,.$$

Exercise 2.3 Secretary Matching *with approximation* (5 + 5 points)

Let *GreedyAlgo* be defined as in the previous exercise and recall that it computes a 2-approximation for the offline weighted Matching problem. Consider a variation of the algorithm for Secretary Matching (algorithm 2) discussed in the lecture where line 8 is replaced by applying *GreedyAlgo* to compute a matching $M^{g,t}$ of $G_t = (L_t \cup R, E_t)$ in each round $t \geq s + 1$.

a) Show that Lemma 1 takes the following form given the modification described above:
For every given round $t = s + 1, \dots, n$, *we have* $\mathbb{E}[v(e_t)] \geq v(M^*)/(2 \cdot n)$.

b) Show that there is a $2 \cdot (e + o(1))$-competitive algorithm for Secretary Matching.

Hint: You can use the result from subtask a). What about Lemma 2?

Commentary: This proof can be extended to arbitrary deterministic α-approximation algorithms for offline weighted Matching, *where $\alpha > 1$. In this way, one obtains an $\alpha \cdot (e + o(1))$-competitive algorithm for* Secretary Matching.

The assignments and further information on the course are provided on our website:
http://algo.cs.uni-frankfurt.de/lehre/oau/sommer2021/oau21.shtml

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