Optimization and Uncertainty

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Assignment 1

Issued: 18.04.2023
Due: 25.04.2023, **10:00h**

- Solutions have to be submitted as **one PDF-file via an individual submission URL**.
- Follow the instructions on our website to sign up and **receive** a submission URL.
- Make sure that your submission reaches us **before 10:00h**.
- Submissions can be composed in English or German.
- Your submission must include your **name** and **matriculation number**.
- In general, you need to **explain** your answers if not stated otherwise.
- This assignment will be discussed in the exercise session on April 28th.

Have a great semester!

Exercise 1.1 *Lower bounds for* OnlineMax (4 points)

Consider the OnlineMax problem discussed in the lecture. Show that no deterministic algorithm is α-competitive, for any finite α.

Exercise 1.2 *Online*-Secretary $(1+2+2+2=7$ points)

Consider the optimal algorithm for the Secretary problem from the lecture. Suppose there are 150 dates in total. After seeing the first 40 people, the first person is accepted whose value exceeds the highest value among the first 40 people. If there is no such person, the last person is accepted.

a) Determine the probability that the best person shows up last.

b) Determine the probability that the best person shows up last **and** that this person is accepted.

c) Determine the probability that the last person is accepted.

d) Determine the **conditional probability** that the last person is accepted given that this is the best person.

Exercise 1.3 *Strategies for the* Secretary *problem* $(3+3=6$ points)

Compute the probability of accepting the best person of the two following strategies for the Secretary problem with n dates.

a) Accept person at position $n-1$ if this person has a higher value than all previous ones. Otherwise accept the last person.

b) Accept person at position $n-2$ if this person has a higher value than all previous ones. Otherwise accept the last person.

Exercise 1.4 *Constant probability in the* SECRETARY *problem* (3 points)

Let $k \in \mathbb{N}_{>0}$ be a fixed constant. Design an online algorithm that accepts the k best people in the SECRETARY problem with constant probability. Assume that the total number of dates n is at least $2k$. Depict your algorithm and show that the probability to accept the k best people is constant for any fixed constant k.