

Exercise 1

Issued: 28.04.2020
Due: 05.05.2020, 14:15h

Please submit your solution in PDF format by sending an email to tds20@cs.uni-frankfurt.de.
Make sure that your solution reaches us before 2:15 pm!

You can compose your solution in English or German.

Exercise 1.1. *Graph Terminology* (6 = 2 + 2 + 2 Points)

- Give an example of a (small) tree with a vertex whose **level**, **min-level**, and **max-level** are three different numbers.
- Let $G = (V, E)$ be an undirected connected graph. Prove that $Diam(G) \leq 2Rad(G)$.
- Let $G = (V, E)$ be an undirected connected graph. Prove or disprove the following statement:
For each pair of centers u, v of G , $dist_G(u, v) \leq 2$.

Exercise 1.2. *Tree Construction* (7 Points)

Consider the synchronous CONGEST-model. In the lecture, we have seen the "FLOOD&ECHO"-procedure which constructs a BFS-tree with prespecified root r_0 . At the end of this procedure, r_0 becomes aware that the construction is complete.

Describe a distributed algorithm that constructs a rooted BFS-tree without being given an explicit root node to begin with. The resulting root node of this constructed tree should be made aware of the completion of the process.

Provide tight asymptotic bounds for time and message complexity in terms of $|E|$ and D , where $|E|$ is the number of edges and D is the diameter of the graph.