Theory of Distributed Systems

Summer Term 2020

Prof. Dr. Martin Hoefer
Marco Schmalhofer

JOHANN WOLFGANG GOETHE
UNIVERSITÄT
FRANKFURT AM MAIN

Institute of Computer Science
Algorithms und Complexity

Exercise 2

Issued: 05.05.2020
Due: 12.05.2020, **14:15h**

Exercise 2.1. *Individual Messages* (8 = 6 + 2 Points)

Consider a network $G = (V, E)$, where $V = \{v_1, \dots, v_n\}$. In the following, let D denote the diameter of G. The **individual messages** (IM) task requires vertex v_1 to deliver a (distinct) $O(\log n)$-bit message to every other vertex in the network along some prespecified shortest route. Assume that shortest routes are precomputed and stored in routing tables, i.e., each vertex v_i knows the next vertex on some shortest path from itself to v_j, for all $j \neq i$.

a) Consider the synchronous CONGEST-model. Prove or disprove each of the following claims regarding the time complexity of the problem.

(i) Time(IM) $= O(D)$ (i.e. there exists a constant $c > 0$ such that for every network G as above, Time(IM,G) $\leq cD$).

(ii) Time(IM) $= O(n)$ (i.e. there exists a constant $c > 0$ such that for every network G as above, Time(IM,G) $\leq cn$).

(iii) Time(IM) $= \Omega(n)$ (i.e. there exists a constant $c > 0$ such that for every $n \geq 1$, there exists an n-vertex network G as above for which Time(IM,G) $\geq cn$).

(iv) There exists a constant $c > 0$ such that for every network G as above, Time(IM,G) $\geq cD$.

b) Now, consider the synchronous LOCAL-model. Prove or disprove Time(IM) $= O(D)$ (i.e. there exists a constant $c > 0$ such that for every network G as above, Time(IM,G) $\leq cD$).

Exercise 2.2. *Route-Disjoint Matching* (5 Points)

Prove the following result from the lecture (also, see Lemma 4.3.2 in the Peleg-Book):

For every tree T with n nodes and every subset W of nodes with $|W| = 2k \leq n$, there exists a route-disjoint matching. The matching can be found by a distributed algorithm on T in time $O(Depth(T))$.

Please turn over!

Exercise 2.3. *Token Distribution* (5 Points)

Prove the following result from the lecture (also, see Lemma 4.3.3 in the Peleg-Book):

For every tree T and every node u, let s_u and n_u be the total number of tokens and nodes in subtree T_u, respectively. There exists a distributed algorithm for performing token distribution on a tree using an optimal number of messages $P = \sum_{u \neq r_0} |s_u - n_u|$ and $O(n)$ time, after a preprocessing stage requiring $O(Depth(T))$ time and $O(n)$ messages.

Exercise 2.4. *Bellman-Ford* (5 Points)

Modify the Bellman-Ford algorithm so that it detects its termination. Make sure that the given bounds on time and message complexity still hold.

E-mail: `mhoefer@cs.uni-frankfurt.de`, `schmalhofer@em.uni-frankfurt.de`