Theory of Distributed Systems

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Exercise 5

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Exercise 5.1. *ThreeColor2Matching on Trees* (4 Points)

Given a legal 3-coloring on a rooted tree, show that it is possible to find a maximal matching in time $O(1)$.

Exercise 5.2. *MIS on 3-regular graphs* ($8 = 2 + 3 + 3$ Points)

A graph is called *3-regular* if each node has a degree of 3. Consider the following distributed algorithm for MIS on 3-regular graphs with n vertices:

In phase $i = 1, 2, \ldots$, each undecided node v does the following: First, it marks itself with probability 1/4. Then, if v is marked and none of its neighbors are marked, v decides to join the independent set and all its neighbors decide to stay out.

Prove the following claims:

a) The expected number of undecided nodes decreases by at least a constant factor in each phase.

b) With high probability (w.h.p.), after $O(\log n)$ phases each node has decided.

c) In expectation it takes $O(\log n)$ phases until every node has decided.

Hint: Markov Inequality and Chernoff Bounds may help.

Exercise 5.3. *Blue-Red Edges* (6 Points)

Consider a weighted graph $G = (V, E, \omega)$ with distinct edge weights. Recall the following from the lecture:

- An edge e is a red edge if there is a cycle in G and e has the highest weight on that cycle.
- An edge e is a blue edge if there is a fragment of the MST T^* such that e has minimum weight among all outgoing edges of the fragment.

Show that there cannot be a single edge that is both red and blue.

Exercise 5.4. *MST on complete graphs* (6 Points)

Show that there is a distributed algorithm to construct an MST on a complete graph with n vertices in $O(\log n)$ rounds. Note that the nodes are aware of the fact that they are in a complete graph.

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