Theory of Distributed Systems

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Exercise 7

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Exercise 7.1. *Function Routing* (3 + 3 = 6 Points)

A *function routing problem* on a graph $G = (V, E)$ is defined by a function $f : V \to V$. It describes the task that, for every $v \in V$, a message of size $O(\log |V|)$ should be sent from v to $f(v)$. Consider the synchronous CONGEST model. Let D be the dilation of the chosen collection of paths. You can assume that dimension-by-dimension is used for the path selection.

a) Show that the time complexity for the function routing problem on $M(\ell, 1)$ is $O(D)$, i.e., there is an algorithm that delivers all messages as described by any function $f : V \to V$ in $O(D)$ steps.

b) Show that the time complexity for the function routing problem on $M(\ell, 2)$ is $\Omega(D^2)$, i.e., there is a function $f : V \to V$ such that any algorithm needs $\Omega(D^2)$ steps to deliver all messages.

Exercise 7.2. *Dimension-By-Dimension Message Routing* (3 + 5 = 8 Points)

How many steps, as a function of the dilation D, does dimension-by-dimension permutation routing require on the mesh $M(\ell, 3)$ ...

a) ... in the synchronous LOCAL-model?

b) ... in the synchronous CONGEST-model? Show a lower as well as an upper bound.

In both cases, we are interested in the worst-case running time of the algorithm, i.e., the time complexity over all possible permutations. Give the bounds in asymptotic notation. Specify your packet scheduling policy.

Exercise 7.3. *Indirect Networks* (5 Points)

Let $G = (V, E)$ be a graph with two special subsets $I \subseteq V$ and $O \subseteq V$ called the inputs and the outputs, respectively. Suppose $|I| = |O|$. Such a network is called *indirect network*. A path system $\mathcal{W}$ for an indirect network contains a path $P_{u,v}$ from every input $u \in I$ to every output $v \in O$. A permutation routing problem on indirect networks is given by a bijective function $\pi : I \to O$ (rather than a complete permutation $\pi : V \to V$).

Generalize the lower bound of Theorem 23 towards permutation routing problems on indirect networks. In particular, prove a lower bound in terms of n, Δ, and r, where $n = |I|$, Δ is the maximum degree of G and r denotes the ratio between the number of nodes and the number of inputs, i.e., $r = |V|/n$. For $r = 1$, your bound should be identical to the one in Theorem 23.

Please turn over!

Exercise 7.4. *Hamiltonian Paths* (4 Points)

Prove or disprove:
The mesh $M(\ell, d)$ has a Hamiltonian path for every ℓ, d.

Reminder: A Hamiltonian path in a graph $G = (V, E)$ is a path in G that contains every vertex in V exactly once.

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