Theory of Distributed Systems

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Exercise 1

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Please submit your solution before the start of Tuesday's lecture at 2:15 pm. You can also use the letter box located between room 114 and room 115. Please write your full name and student id on the solution. If you need several pages, please staple them together.

You can compose your solution in English or German.

Exercise 1.1. *Tree Levels* (3 Points)

Give an example of a (small) tree with a vertex whose **level**, **min-level**, and **max-level** are three different numbers.

Exercise 1.2. *Route-Disjoint Matching* (5 Points)

Prove the following result from the lecture (also, see Lemma 4.3.2 in the Peleg-Book):

For every tree T with n nodes and every subset W of nodes with $|W| = 2k \leq n$, there exists a route-disjoint matching. The matching can be found by a distributed algorithm on T in time $O(Depth(T))$.

Exercise 1.3. *Token Distribution* (5 Points)

Prove the following result from the lecture (also, see Lemma 4.3.3 in the Peleg-Book):

For every tree T and every node u, let s_u and n_u be the total number of tokens and nodes in subtree T_u, respectively. There exists a distributed algorithm for performing token distribution on a tree using an optimal number of messages $P = \sum_{u \neq r_0} |s_u - n_u|$ and $O(n)$ time, after a preprocessing stage requiring $O(Depth(T))$ time and $O(n)$ messages.

Exercise 1.4. *Tree Construction* (7 Points)

Consider the synchronous $\mathcal{CONGEST}$-model. In the lecture, we have seen the "Flood&Echo"-procedure which constructs a BFS-tree with prespecified root r_0. At the end of this procedure, r_0 becomes aware that the construction is complete.

Describe a distributed algorithm that constructs a rooted BFS-tree without being given an explicit root node to begin with. The resulting root node of this constructed tree should be made aware of the completion of the process.

Provide tight asymptotic bounds for time and message complexity in terms of $|E|$ and D, where $|E|$ is the number of edges and D is the diameter of the graph.

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