Theory of Distributed Systems

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Exercise 5

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For all tasks we consider the CONGEST-model. This means that messages are of size $O(\log n)$.

Further, we consider connected, weighted graphs $G = (V, E, \omega)$. Note that the weights do not have to be distinct.

Consider the following variant of the Mailing Problem, the **Orthogonal Mailing Problem**:

Given a graph G with two specified nodes $s \neq r$ as well as bit-vectors $b^{(s)}$ and $b^{(r)}$ of size k for s and r, respectively. Find out whether the bit vectors are orthogonal, i.e., r wants to find out if $\sum_{i=1}^{k} b_i^{(s)} b_i^{(r)} = 0$.

Lemma:
For every $m \geq 1$, the Orthogonal Mailing Problem for $k = m^2$ cannot be solved in time $o(m^2/\log m)$ on the hard graph HG_m by a distributed algorithm.

Exercise 5.1. *Weighted Distances* (6 Points)

Use the above lemma to show that in the class of hard graphs finding any approximation to the weighted distance between a pair of nodes $s, t \in V$ takes $\Omega(\sqrt{n}/\log n)$ rounds.

Exercise 5.2. *Weighted Cuts* (6 Points)

For $s, t \in V$, an s-t-cut is a subset $S \subseteq V$ with $s \in S$ and $t \notin S$. The weight of the cut is the sum of the weights of all edges $\{v, w\} \in E \cap (S \times (V \setminus S))$, i.e. the edges crossing the cut.

Use the above lemma to show that in the class of hard graphs finding any approximation to the weight of a minimum s-t-cut takes $\Omega(\sqrt{n}/\log n)$ rounds.

Exercise 5.3. *More MSTs on Rings* (4 Points)

Give a deterministic distributed algorithm for MST construction on an n-vertex, non-anonymous weighted ring with a given initiator node. Note that the weights do not have to be distinct.

By the end of the algorithm, each vertex should know its parent and its child in the MST. The algorithm should take at most n (not $O(n)$) many time units.

Exercise 5.4. *Dual Greedy Message Complexity* (2 + 3 = 5 Points)

Derive upper bounds in terms of n, $Diam(G)$, and $|E|$ on the message complexity of the following parts of the Dual Greedy algorithm. Give example graphs to show that your bounds are tight.

a) Construction of the BFS tree
b) Pipelined upcast of edges