Theory of Distributed Systems

Winter Term 2018/2019

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Exercise 8

Issued: 18.12.2018
Due: 22.01.2019

Exercise 8.1. *ℓ Pushes* (3 + 3 = 6 Points)

Adapt the Degree-Diameter Bound (Theorem 29) to the following protocol:

Instead of sending the rumor to a single neighbor in each of the rounds, send out $\ell \leq \Delta$ messages where Δ is the maximum degree of the graph. Analyze the following two variants:

a) In each round independently, ℓ different neighbors are picked (each subset with equal probability). In the case that a node only has less then ℓ neighbors, it just sends the rumor to all its neighbors.

b) In each round independently, each node that has the rumor repeats the following process ℓ times: The node picks a neighbor uniformly at random and pushes the message to this neighbor.

Exercise 8.2. *ℓ Rumors* (5 Points)

Consider the following scenario: Instead of a single rumor that all nodes need to learn, there are ℓ different rumors. Each rumor starts at some node. The Push protocol is used.

Bound the number of rounds required until a given node v knows all rumors with probability at least $1 - 1/n$ in terms of $n, \Delta, L,$ and ℓ. Here, n is the number of nodes in the graph, Δ is the maximum degree, and L is the maximum length of a shortest path from v to a node which knows a rumor at the start.

Exercise 8.3. *Pull on a Star Graph* (3 + 3 = 6 Points)

Consider the Pull protocol on a star graph with n nodes. Under the assumption that the rumor starts at one of the satellites, show the following:

a) Show $\mathbb{E}[T_{all}] = O(n)$, where T_{all} is the time until all nodes have been informed.

b) With probability at least $1 - 1/n$, all nodes are informed in $O(n \log n)$ rounds.

Please turn over!

Exercise 8.4. *Lower Bound for the Pull Protocol* (4 Points)

In the lecture, we showed Theorem 28: "Let T be an integer such that in round T of the Push protocol with probability $1 - 1/n$ every node is informed. For every connected graph G it holds $T = O(n \log n)$ and $T = \Omega(\log n)$."

On the last sheet, we showed that the lower bound is actually a deterministic one, meaning that the probability for T being in $o(\log n)$ is 0 for all graphs.

Consider the Pull protocol. Does a similar lower bound hold? Explain your answer.

We wish everyone happy holidays and great start into 2019!

The assignments and further information concerning the lecture can be found at
`http://algo.cs.uni-frankfurt.de/lehre/tds/winter1819/tds1819.shtml`

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