# Theory of Distributed Systems

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# Exercise 9

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**Exercise 9.1.** *ALOHA* (4 Points)

Show Lemma 61 from the notes:

The ALOHA protocol elects a leader in $O(\log n)$ rounds w.h.p.

**Exercise 9.2.** *ULE-CD* (5 Points)

Show Theorem 33 from the notes:

Algorithm ULE-CD is a uniform algorithm for leader election with collision detection that runs in time $O(\log n)$ w.h.p.

*Hint: Derive a bound on the expected number of nodes that are present after $O(\log n)$ rounds. How does the result follow from that?*

**Exercise 9.3.** *Max-Average-Degree* (4 Points)

Give an example of a graph $G$ with $n$ nodes with $MaxAvg(G) = \Theta(n)$ and $\chi(G) = O(1)$. Here, $\chi(G)$ is the chromatic number of the graph.

**Exercise 9.4.** *Disk-Graphs on the Line* (3 + 3 = 6 Points)

Suppose there are $n$ base stations located along a line. Each base station tries to reach mobile receivers in the vicinity on the line.

Formally, we assume for each base station $i$ there is a continuous line segment of length $\ell_i > 0$ and the base station is located in the middle of this segment. Two base stations are conflict-free if and only if their segments do not intersect.

The resulting conflict graph $G$ can be seen as a "one-dimensional disk-graph". Show the following for the inductive independence number $\rho(G)$:

a) $\rho(G) \leq 2$

b) $\rho(G) \leq 1$

*Hint: Obviously, a solution to b) is sufficient to solve a) as well.*



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