Theory of Distributed Systems

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Exercise 5

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Due: 30.11.2021, **8:15h**

Please submit your solution in PDF format by sending an email to {schmalhofer,varricchio}@em.uni-frankfurt.de. Make sure that your solution reaches us before 8:15 am! Solutions are discussed on Dec 3rd, 10:00h - 12:00h (Zoom Meeting-ID: 963 6309 6725, same password as lecture material).
Exercises with * are bonus; they count for your score but not for the sum of points.

Exercise 5.1. *Lower Bounds for MST Algorithms* (5 = 2 + 3 Points)

a) Provide an n-vertex edge-weighted network $G = (V, E, \omega)$ such that the running time of the Dual Greedy Algorithm is $\Omega(n)$.

b) Provide an n-vertex edge-weighted network $G = (V, E, \omega)$ such that the running time of the GHS Algorithm is $\Omega(n \log n)$.

Exercise 5.2. *MST on Complete Graphs* (5 Points)

Show that there is a distributed algorithm to construct an MST on a complete graph with n vertices in $O(\log n)$ rounds. Note that the nodes are aware of the fact that they are in a complete graph.

Exercise 5.3. *MST and Leader Election on Rings* (6 = 3 + 3 and 5* Points)

In leader election, we want to determine the node with maximum ID in the network. This node will be called the *leader*. At the end of the leader election process, the elected node must be aware of being the leader, while all the other nodes know they are not the leader.

Consider an unweighted ring network $R = (V, E)$, where each one of the n nodes has a distinct ID. In this exercise, you can always assume that it is not possible to a priori select a starting node r_0.

a) Show that if there exists a distributed MST algorithm (for possibly non-distinct weights) with time complexity $T(G)$, then it is possible to perform leader election on the ring R in time $T(R) + O(1)$.

b) Show that if there exists a leader election algorithm with time complexity $T(R)$, then, for any function ω of distinct edge weights, it is possible to compute an MST in time $T(R) + O(1)$.

c*) Provide an algorithm that computes an MST on rings with distinct edge weights. Time and message complexity should be $O(n)$ and $O(n \log n)$, respectively.

Exercise 5.4. *Non-distinct Weights* (6 = 4 + 2 Points)

Consider an edge-weighted network $G = (V, E, \omega)$ with n nodes and unique IDs from $\{0, \ldots, n-1\}$. The (positive and integer) weights of different edges might be identical; each weight is encoded by $O(\log n)$ bits.

a) Define a new edge weight $\omega'(e)$ for each edge $e \in E$, based on only its original weight $\omega(e)$ and the IDs of its endpoints, such that all the following properties hold:

   i) for each pair $e \neq f$ of edges, $\omega'(e) \neq \omega'(f)$,

   ii) the new weights can be encoded using $O(\log n)$ bits, and

   iii) every MST in $G' = (V, E, \omega')$ is also an MST in $G = (V, E, \omega)$.

b) Show that the new weights you defined satisfy i), ii), and iii).

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