Theory of Distributed Systems

Winter Term 2021/22

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Exercise 9

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Due: 25.01.2022, **8:15h**

Please submit your solution in PDF format by sending an email to {schmalhofer,varricchio}@em.uni-frankfurt.de. Make sure that your solution reaches us before 8:15 am! Solutions are discussed on Jan 28th, 10:00h - 12:00h (Zoom Meeting-ID: 963 6309 6725, same password as lecture material).

Exercise 9.1. *ALOHA* (4 Points)

Show Lemma 61 from the notes:

The Slotted ALOHA protocol elects a leader in $O(\log n)$ rounds whp.

Exercise 9.2. *k-Channel Leader Election* (7 = 3 + 4 Points)

Consider a wireless network with n nodes. There are $k = \sqrt{n}$ independent channels numbered from 1 to $\sqrt{n}$. In each round t, each node j chooses one channel k_j^t. Then j can transmit or listen in round t only on k_j^t. Transmissions of two or more nodes on distinct channels do not interfere, on the same channel they yield a collision. Nodes do not have collision detection.

Suppose we want to elect a unique leader for each channel.

a) Prove that the following algorithm needs $O(\sqrt{n}\log n)$ time steps whp:

   In phase $i = 1 \dots \sqrt{n}$, every remaining device runs the Slotted ALOHA protocol on channel i until a leader for channel i is elected. The leader for channel i gets informed and drops out. The transmitting probability is always fixed to $1/n$.

b) Consider the following algorithm:

   In the beginning, each node selects a single channel uniformly at random. Then each node runs Slotted ALOHA only on its selected channel. If in the end there is a channel without leader, the algorithm restarts. The transmitting probability is always fixed to $1/\sqrt{n}$.

   Show that the algorithm needs $O(\log n)$ time steps whp.

Exercise 9.3. *Transmission Complexity* (7 = 3 + 4 Points)

Consider again a wireless network with n nodes. A *transmission attempt* is a single try of a single node to transmit a message on the medium. The overall number of transmission attempts of an algorithm is called *transmission complexity*.

Consider the algorithm InitCD from the lecture.

a) Show that *there is* an execution of InitCD, where there are $\Omega(n^2)$ transmission attempts.

b) Show that *in every* execution of InitCD, there are at least $\Omega(n \log n)$ transmission attempts.

Exercise 9.4. *Estimating the Number of Nodes* (4 Points)

Consider the following algorithm for estimating the number of nodes in a wireless network: Replace line 15 of fast-ULE-CD through "Output $\hat{n} := 2^u$ and terminate". The new algorithm outputs an estimate $\hat{n}$ for the number of nodes n.

Prove or disprove the following confidence guarantee on $\hat{n}$:

For sufficiently large n it holds

$$\Pr[\, n/2 \leq \hat{n} \leq 2n \,] \geq 99\%.$$

Hint: You can think of specific executions which happen with constant probability for large n.

The assignments and further information concerning the lecture can be found at
`http://algo.cs.uni-frankfurt.de/lehre/tds/winter2122/tds2122.shtml`

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